Electric Potential at a point in an electric field is defined as the work done per unit charge in bringing a unit positive charge from infinity to that point without any acceleration and against the electric force. $V = \frac{W_{\infty \to r}}{q_0}$

Electric Potential Difference between two points in an electric field is defined as the work done per unit charge in bringing a positive test charge from one point to another without any acceleration against the electric force. $V_{AB} = \frac{W_{A \to B}}{q_0}$

Unit of electric potential and potential difference is volt (V).

Dimensions of Potential

$$[V] = \frac{[W]}{[q]} = \frac{M^1 L^2 T^{-2}}{A^1 T^1} = M^1 L^2 T^{-3} A^{-1}$$

Define 1 volt

The potential at a point in an electric field is said to be 1 volt if 1 joule of work is done in bringing unit positive charge (1 C) from infinity to that point against the electric field without any acceleration.

$$1\ volt = \frac{1\ joule}{1\ coulomb}$$

The potential difference between two points in an electric field is said to be 1 volt if 1 joule of work is done in moving unit positive charge (1 C) from one point to the other against the electric field without any acceleration.

Electric Potential due to a point charge

Consider a point charge q kept at a point O. Let P be a point at a distance r from O where the electric potential is to be calculated.

Let a point test charge q_0 be kept at P. The force acting on q_0, $F = \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r^2}$

The work done by an external agent against the repulsive force between the two charges through a small distance dr,

$$dW = -F dr = -\frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r^2} dr$$

The total work done by the external agency in moving the charge q_0 from infinity to the point P,

$$W = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r^2} dr = -\frac{q q_0}{4\pi\varepsilon_0} \left[\frac{-1}{r}\right]_{\infty}^{r}$$

$$= \frac{q q_0}{4\pi\varepsilon_0} \left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r}$$

By definition of potential, $V = \frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

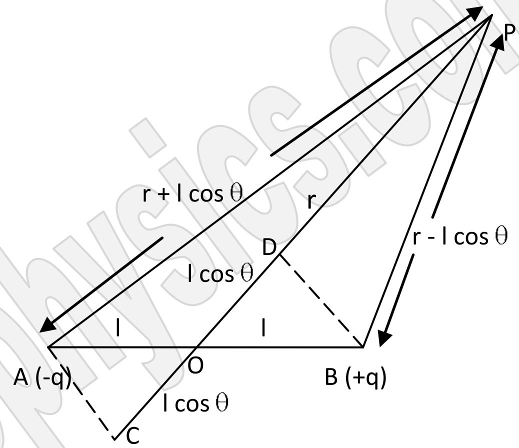
Electric potential due to a system of charges is the algebraic sum of potentials due to individual charges.

If charges q_1, q_2, q_3 … q_N are located at distances r_1, r_2, r_3, … r_N from a point P, then the resultant potential at P,

$$V = V_1 + V_2 + V_3 + \ \ldots + V_N = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i}$$

Electric Potential due to a dipole

Let AB be a dipole of dipole length 2l formed by two point charges -q and +q kept at A and B respectively as in figure. Let P be a point at a distance r from the centre O of the dipole.



The electric potential due to dipole at the point P,

V = V_1 due to –q + V_2 due to +q

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{AP} + \frac{q}{BP}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{-1}{(r + l\cos\theta)} + \frac{1}{(r - l\cos\theta)}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{r + l\cos\theta - r + l\cos\theta}{(r^2 - l^2\cos^2\theta)}\right) = \frac{q}{4\pi\varepsilon_0}\left(\frac{2l\cos\theta}{(r^2 - l^2\cos^2\theta)}\right) = \frac{1}{4\pi\varepsilon_0} \frac{P\cos\theta}{(r^2 - l^2\cos^2\theta)}$$

If the dipole is very short (r >> l), then

$$V = \frac{1}{4\pi\varepsilon_0} \frac{P\cos\theta}{r^2}$$

If P is on the axial line, θ=0° and $V_{axial} = \frac{1}{4\pi\varepsilon_0} \frac{P}{r^2}$

If P is on the equatorial line, θ=90° & $V_{equatorial}$ = 0

Electric Potential due to a uniformly charged thin spherical shell

Consider a charged spherical conducting shell of radius R

(a) at a point outside the shell, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r}$

(b) at a point on the shell, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$

(c) At a point inside the shell, $V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}$

Relation between electric field and potential

When a charge q_0 is kept in an electric field E, the force acting on it, $F=q_0E$

The work done by the external agency in moving the test charge through a small distance dr against the field, $dW = -F\,dr = -(q_0E)dr$

OR $\frac{dW}{q_0} = -Edr$

But $\frac{dW}{q_0} = V \;\therefore V = -Edr$ OR $E = -\frac{dV}{dr}$

That is **electric field is the negative gradient of potential**.

Also; The potential, $V = -\int_{\infty}^{r} \vec{E}.\overrightarrow{dr}$

Potential difference between two points A and B

$$V_{AB} = -\int_{r_A}^{r_B} \vec{E}.\overrightarrow{dr}$$

Equipotential Surface is that surface on which every point will have the same potential.

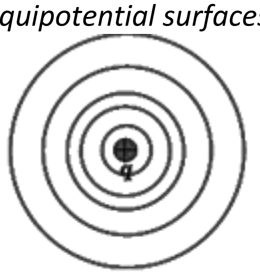
OR Equipotential surface is the locus of all points which are at the same potential

Properties of Equipotential Surfaces

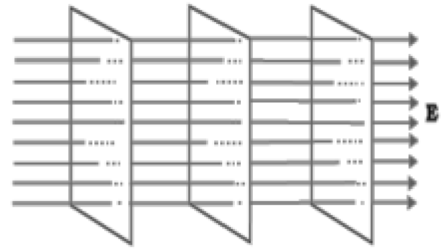
1. *Potential at every point on an Equipotential surface is the same*
2. *No work is done in moving a charge along an equipotential surface*
3. *The electric field lines are always normal to an equipotential surface.*
4. *Equipotential surfaces intersect each other. (If they intersect, two normals can be drawn at any point on the region of intersection)*
5. *The relative separation between equipotential surfaces is a measure of the intensity of electric field in that region. (If the equipotential surfaces drawn with constant pd between them are closer to each other the electric field there is stronger and vice versa)*

Shape of equipotential surfaces due to various charge systems

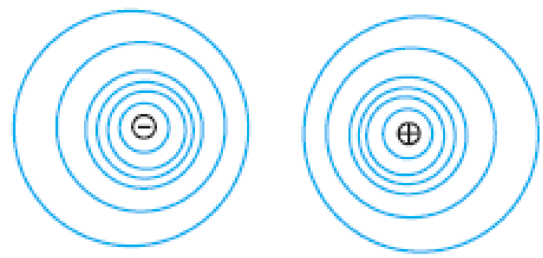
1. *Equipotential surfaces due to a point charge*



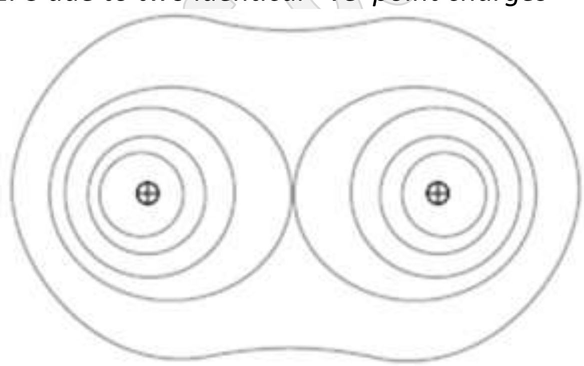
2. *EPS in a uniform electric field*



3. *EPS due to a dipole*



4. *EPS due to two identical +ve point charges*



Electrostatic Potential Energy

Electrostatic Potential Energy of a system of charges is defined as the total work done in bringing the charges to their respective positions from infinity against the electric forces without any acceleration.

System of two charges

Let a charge q_1 be brought to a field free space from infinity. The work done in doing so, $W_1=0$ (since there is no other charge or field to oppose it). Let another charge q_2 be brought to a point at a distance r_{12} from q_1. The external agency must do some work for this. The potential at the position of q_2 due to the charge q_1, $\boldsymbol{V_{21}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}}$

The work done, $\mathbf{W_2=q_2\,V_{21}} = q_2\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$

System of three charges

(Proceed as above and calculate W_1 and W_2)

Let a third charge q_3 be brought at a point distant r_{13} from q_1 and r_{23} from q_2. Let the potential at the point due to q_1 be $V_{31} = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}}$ and due to q_2 be $V_{32} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}$

Therefore, The work done in bringing q_3,

$$W_3 = q_3(V_{31} + V_{32}) = q_3\left(\frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{23}}\right) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Therefore the Potential energy of the system of three charges $U = W_1 + W_2 + W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$

System of N charges: Proceeding as above, the potential energy of a system of N charges can be written as

$$U = \frac{1}{4\pi\varepsilon_0}\frac{1}{2}\sum_{\substack{i=1\\j=1\\i\neq j}}^{N}\frac{q_i q_j}{r_{ij}}$$

Potential energy of a point charge in an external electric field is given by U = qV

Potential energy of a dipole in a uniform electric field

When a dipole is brought with its axis making an angle 90° with the electric field, the work done will be zero since whatever work done on the negative charge will be equal and opposite to the work done on the positive charge. Therefore the potential energy of the dipole at any orientation θ will be equal to the work done in rotating the dipole from 90° to θ

$\therefore U_\theta = W_{90° \to \theta} = -PE(\cos\theta - \cos 90) = -pE\cos\theta$

In vector form $U_\theta = -\vec{P}.\vec{E}$

Conductors and insulators

The substances which conduct electricity are called conductors. Metals are good conductors as they have plenty of free electrons. (About 10^{29} m^{-3})

Insulators do not conduct electricity as they do not have any free mobile charges in them.

Free and bound charges

The charges which are attracted strongly by the nucleus cannot be isolated easily and they are called bound charges. The electrons of the outermost shell can be easily removed and they are relatively free. In the case of metals each atom of the metal is surrounded by a sea of electrons and the electrons can be easily dislodged.

If a charged body is brought near a conductor, the near side gets induced with opposite charges and the far side with like charge as the original charged body. If the conductor is earthed, the charge at the far end will get discharged (whatever be the portion of the conductor connected to earth) and the charge near the original charged body cannot move as they are bound by the force of attraction.

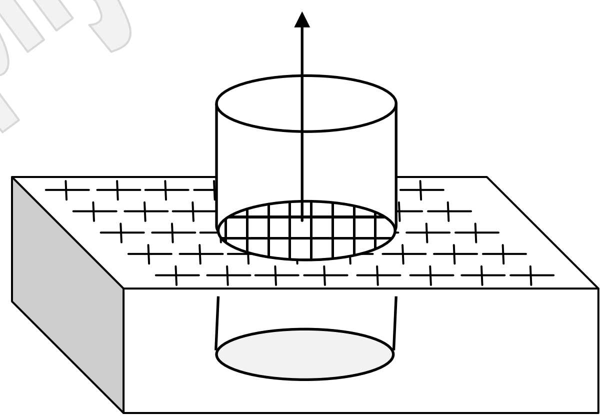
Behavior of conductors in external electric fields

When a conductor is kept in an external electric field, the free charges will flow in such a way that the electric field produced by the displacement of free charges is equal to the external field but in opposite direction and therefore the NET ELECTRIC FIELD INSIDE A CONDUCTOR IS ALWAYS ZERO.

Show that the electric field near a charged conductor is equal to σ/ε_0

Consider a thick conductor of surface charge density σ

Imagine a point very close to its surface. Imagine a Gaussian cylinder passing through the point with one face inside the conductor. Since the electric field inside the conductor is zero, there will be flux



coming out only through the outer face. Applying Gauss theorem, $\oint \vec{E}\,.\,\vec{dS} = \frac{q}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0}$

$$ES = \frac{\sigma S}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

This is true for any charged conductor.