SOLVED EXAMPLE

Example 1: Assuming earth to be an isolated conducting sphere of radius 6400 km, what is the capacitance of earth?

Solution: Capacitance of earth, $C = 4\pi\varepsilon_0 r$

Here, $4\pi\varepsilon_0 = \dfrac{1}{9\times10^9}$; $r = 6400\text{ km} = 6.4\times10^6\text{ m}$

$$\therefore \quad C = \frac{6.4\times10^6}{9\times10^9} = 0.711\times10^{-3}\text{ F} = \mathbf{711\ \mu F}$$

This shows that farad is a very large unit of capacitance.

Note: Since capacitance of earth is quite large, we choose earth as a level of zero potential for practical purposes. Think about this !

Example 2. *An isolated sphere has a capacitance of 50pF. (i) Calculate its radius. (ii) How much charge should be placed on it to raise its potential to 10^4V?*

Solution:

(*i*) Capacitance of sphere, $C = 4\pi\varepsilon_0 r$

$$\therefore \text{ Radius of sphere, } r = \frac{1}{4\pi\varepsilon_0}\times C = (9\times10^9)\times(50\times10^{-12}) = 45\times10^{-2}\text{ m} = \mathbf{45\,cm}$$

(*ii*) Charge to be placed, $q = CV = (50\times10^{-12})\times10^4 = 5\times10^{-7}\ C = \mathbf{0.5\ \mu C}$

Example 3. *Twenty seven spherical drops, each of radius 3mm and carrying 10^{-12}C of charge are combined to form a single drop. Find the capacitance and potential of the bigger drop.*

Solution. Let r and R be the radii of smaller and bigger drops respectively.

Volume of bigger drop = 27 × Volume of smaller drop

or $$\frac{4}{3}\pi R^3 = 27\times\frac{4}{3}\pi r^3$$

or $$R = 3r = 3\times3 = 9\text{ mm} = 9\times10^{-3}\text{ m}$$

Capacitance of bigger drop, $$C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9}\times9\times10^{-3} = 10^{-12}\text{ F} = \mathbf{1\,pF}$$

Since charge is conserved, the charge on the bigger drop is 27×10^{-12} C.

$\therefore$ Potential of bigger drop, $$V = \frac{q}{C} = \frac{27\times10^{-12}}{10^{-12}} = \mathbf{27\,V}$$

Example 4: *A spherical capacitor has an inner sphere of radius 9 cm and an outer sphere of radius 10cm. The outer sphere is earthed. Assume there is air in the space between the spheres. What is the capacitance of the capacitor?*

Solution: Capacitance of the spherical capacitor is

$$C = \frac{4\pi\varepsilon_0 r_A r_B}{(r_B - r_A)}$$

Here $r_B = 10\text{ cm} = 0.1\text{ m}$; $r_A = 9\text{ cm} = 0.09\text{ m}$; $4\pi\varepsilon_0 = \dfrac{1}{9\times10^9}$

$$\therefore \quad C = \frac{0.09\times0.1}{9\times10^9\,(0.1-0.09)} = 100\times10^{-12}\text{ F} = \mathbf{100\ pF}$$

Example 5. *The thickness of air layer between two coatings of a spherical capacitor is 2 cm. The capacitor has the same capacitance as the capacitance of sphere of 1.2m diameter. Find the radii of its surfaces.*

Solution:

Given: $\dfrac{4\pi\varepsilon_0 r_A r_B}{r_B - r_A} = 4\pi\varepsilon_0 R \quad \therefore \dfrac{r_A r_B}{r_B - r_A} = R$

Here $r_B - r_A = 2\text{ cm}$ and $R = 1.2/2 = 0.6\text{m} = 60\text{ cm}$

$\therefore \dfrac{r_A r_B}{2} = 60$ or $r_A r_B = 120\text{ cm}$

Now $(r_B + r_A)^2 = (r_B - r_A)^2 + 4r_A r_B = (2)^2 + 4\times120 = 484$

$$\therefore \quad r_B + r_A = \sqrt{484} = 22\text{ cm}$$

Since $r_B - r_A = 2$cm and $r_B + r_A = 22$cm, $r_B = \mathbf{12cm}$; $r_A = \mathbf{10\ cm}$

Example 6. *The plates of a parallel plate air capacitor are separated by a distance of 1 mm. What must be the plate area if the capacitance of the capacitor is to be 1F?*

Solution: The capacitance of a parallel plate air capacitor is given by;

$$C = \frac{\varepsilon_0 A}{d}$$

Here $d = 1\text{ mm} = 10^{-3}\text{ m}; \quad A = ?; \quad C = 1\text{F}$

$$\therefore \quad A = \frac{Cd}{\varepsilon_0} = \frac{1 \times 10^{-3}}{8.854 \times 10^{-12}} = \mathbf{1.1 \times 10^8\ m^2}$$

Note the enormous magnitude of plate area required to have a capacitance of 1F. This shows that farad is a very large unit of capacitance.

Example 7: *What distance apart should the two plates each of area 0.2 m × 0.1 m of a parallel plate air capacitor be placed in order to have the same capacitance as a spherical conductor of radius 0.5m?*

Solution: Area of plate, $A = 0.2 \times 0.1 = 0.02\text{ m}^2$

Radius of sphere, $r = 0.5\text{ m}$

For parallel plate capacitor, $C = \varepsilon_0 A/d$

For spherical conductor, $C = 4\pi\varepsilon_0 r$

Since the capacitance of the two capacitors is the same,

$$\therefore \quad \frac{\varepsilon_0 A}{d} = 4\pi\varepsilon_0 r$$

or
$$d = \frac{A}{4\pi r} = \frac{0.02}{4\pi \times 0.5} = 3.18 \times 10^{-3}\text{ m} = \mathbf{3.18\ mm}$$

Example 8. *Calculate the capacitance of a parallel plate air capacitor of plate area 30 m2; the plates being separated by a dielectric 2 mm thick and of relative permittivity 6. If the electric field strength between the plates is 500 V/mm, calculate the charge on each plate.*

Solution: Capacitance, $C = \frac{\varepsilon_0 K A}{d} = \frac{(8.854 \times 10^{-12})(6)(30)}{2 \times 10^{-3}} = 0.797 \times 10^{-6}\text{ F} = \mathbf{0.797\ \mu F}$

P.D. across plates, $V = E \times d = 500 \times 2 = 1000$ volts

$\therefore$ Charge on each plate, $q = CV = (0.797 \times 10^{-6})1000 = 0.797 \times 10^{-3}\text{ C} = \mathbf{0.797\ mC}$

Example 9. *A p.d. of 10 kV is applied to the terminals of a capacitor consisting of two parallel plates, each having an area of 0.01 m*^{2} *separated by a dielectric 1 mm thick. The resulting capacitance of the arrangement is 300pF. Calculate (i) charge on each plate, (ii) electric flux density, (iii) potential gradient, and (iv) relative permittivity of the dielectric.*

Solution: $C = 300\text{ pF} = 300 \times 10^{-12}\text{ F}; \quad V = 10\text{ kV} = 10 \times 10^3$ volts

(*i*) Charge on each plate, $q = CV = (300 \times 10^{-12})(10 \times 10^3) = \mathbf{3 \times 10^{-6}\ C}$

(*ii*) Electric flux density, $\sigma = \frac{q}{A} = \frac{3 \times 10^3}{0.01} = \mathbf{3 \times 10^{-4}\ C/m^2}$

(*iii*) Potential gradient, $E = \frac{V}{d} = \frac{10 \times 10^3}{1 \times 10^{-3}} = \mathbf{10^7\ V/m}$

(*iv*) Electric intensity, $E = \frac{\sigma}{\varepsilon_0 K}$

$\therefore$ Relative permittivity, $K = \frac{\sigma}{\varepsilon_0 E} = \frac{3 \times 10^{-4}}{8.854 \times 10^{-12} \times 10^7} = \mathbf{3.39}$

Example 10. *A parallel plate capacitor is to be designed with a voltage rating 1kV using a material of dielectric constant 3 and dielectric strength of 107 Vm*$^{-1}$*. What minimum area of the plate is required to have a capacitance of 50 pF?*

Solution. For reasons of safety, the electric field E between the plates should not exceed 10% of the dielectric strength of the dielectric *i.e.* $E = 10\%$ of $10^7 = 10^6\text{ Vm}^{-1}$.

Now $E = \frac{V}{d} \qquad \therefore \quad d = \frac{V}{d} = \frac{1 \times 10^3}{10^6} = 10^{-3}\text{ m}$

$\therefore$ Plate area $A = \frac{(50 \times 10^{-12}) \times 10^{-3}}{8.85 \times 10^{-12} \times 3} = \mathbf{1.9 \times 10^{-3}\ m^2}$

Example 11. *Two parallel plate air capacitors have their plate areas 100 cm² and 500 cm² respectively. If they have the same charge and potential and the distance between the plates of the first capacitor is 0.5 mm, what is the distance between the plates of the second capacitor?*

Solution: Let us denote the first capacitor by suffix 1 and second capacitor by suffix 2. Since the two capacitors have the same charge and potential, their capacitances ($C = q/V$) are equal *i.e.* $C_1 = C_2$.

$$\therefore \quad \frac{\varepsilon_0 A_1}{d_1} = \frac{\varepsilon_0 A_2}{d_2} \qquad \therefore \quad d_2 = \frac{A_2}{A_1} d_1$$

Here $A_2 = 500 \text{ cm}^2$; $A_1 = 100 \text{ cm}^2$; $d_1 = 0.5 \text{ mm} = 0.05 \text{ cm}$

$$\therefore \quad d_2 = \frac{500}{100} \times 0.05 = \mathbf{0.25\,cm}$$

Example 12. *Three capacitors have capacitances of 0.5 μF, 0.3 μF and 0.2 μF respectively. They are first connected to have maximum capacitance and then connected to have minimum capacitance. Find the ratio of maximum capacitance to minimum capacitance.*

Solution: (*i*) For maximum capacitance, all the capacitors will have to be connected in parallel.

$$C_P = 0.5 + 0.3 + 0.2 = 1\ \mu\text{F}$$

(*ii*) For minimum capacitance, all the capacitors will have to be connected in series.

$$\frac{1}{C_S} = \frac{1}{0.5} + \frac{1}{0.3} + \frac{1}{0.2} = \frac{31}{3}$$

or $\quad C_S = 3/31\ \mu\text{F}$

$$\therefore \quad \frac{C_P}{C_S} = \frac{\mathbf{31}}{\mathbf{3}}$$

Example 13. *Two capacitors of capacitance 15 μF and 20 μF are connected in series to a 600 V d.c. supply. Find (i) charge on each capacitor, (ii) p.d. across each capacitor.*

Solution: (*i*) Equivalent capacitance, $C_S = \dfrac{C_1 C_2}{C_1 + C_2} = \dfrac{15 \times 20}{15 + 20} = 8.57\ \mu\text{F}$

In series connection, charge on each capacitor is the same.

$\therefore$ Charge on each capacitor, $q = C_S V = (8.57 \times 10^{-6}) \times 600 = \mathbf{5.14 \times 10^{-3}\ C}$

(*ii*) P.D. across 15 μF capacitor $= \dfrac{q}{C_1} = \dfrac{5.14 \times 10^{-3}}{15 \times 10^{-6}} = \mathbf{342.7\ V}$

P.D. across 20 μF capacitor $= \dfrac{q}{C_2} = \dfrac{5.14 \times 10^{-3}}{20 \times 10^{-6}} = \mathbf{257\ V}$

Example 14. *The total capacitance of two capacitors is 4μF when connected in series and 18 μF when connected in parallel. Find the capacitance of each capacitor.*

Solution: Let C_1 and C_2 be the unknown capacitances. Then,

$C_1 + C_2 = 18$ ...(*i*) when in parallel

$\dfrac{C_1 C_2}{C_1 + C_2} = 4$ ...(*ii*) when in series

Multiplying eqs. (*i*) and (*ii*), $C_1 C_2 = 72$

Now $C_1 - C_2 = \sqrt{(C_1 + C_2)^2 + 4C_1 C_2} = \sqrt{(18)^2 - 4 \times 72} = \pm 6$ ... (*iii*)

Solving eqs. (*i*) and (*iii*), we get, $\quad$ $\mathbf{C_1 = 12\ \mu F\ or\ 6\ \mu F;\ C_2 = 6\ \mu F\ or\ 12\ \mu F}$

Example 15. In *the circuit shown in Fig. 5.13, the total charge is 750 μC. Find the values of V_1, V and C_2.*

Solution: Voltage across $\quad C_1, V_1 = \dfrac{q}{C_1} = \dfrac{(750 \times 10^{-6})}{15 \times 10^{-6}} = \mathbf{50\ V}$

Applied voltage, $V = V_1 + V_2 = 50 + 20 = \mathbf{70\ V}$

Charge on $C_3 = C_3 V_2 = (8 \times 10^{-6})\, 20 = 160 \times 10^{-6} = 160\ \mu\text{C}$

$\therefore$ Charge on $C_2 = 750 - 160 = 590\ \mu\text{C}$

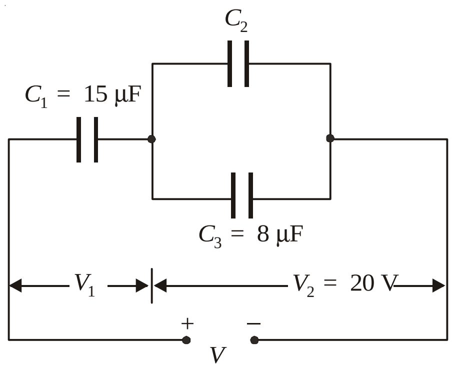


Fig. 5.13

$\therefore$ Capacitance of $C_2 = \dfrac{590 \times 10^{-6}}{20} = 29.5 \times 10^{-6}$ F = **29.5 μF**

Example 16: *Obtain the equivalent capacitance for the network shown in Fig. 5.14. For 300 V d.c. supply, determine the charge and voltage across each capacitor.*

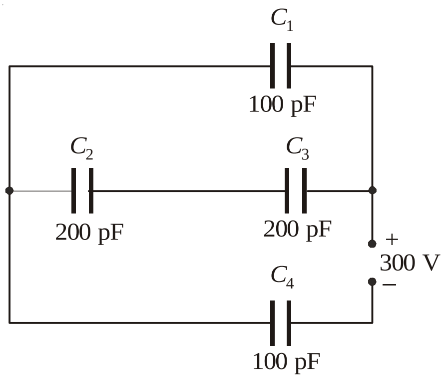


Fig. 5.14

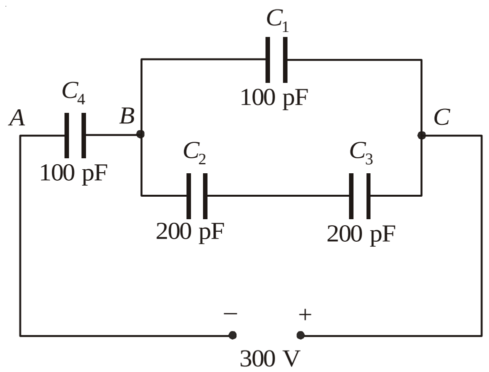


Fig. 5.15

Solution: Equivalent Capacitance. The above network can be redrawn as shown in Fig. 5.15. The equivalent capacitance C' of series-connected capacitors C_2 and C_3 is

$$C' = \frac{C_2 \times C_3}{C_2 + C_3} = \frac{200 \times 200}{200 + 200} = 100\,\text{pF}$$

The equivalent capacitance of parallel combination of C' (= 100 pF) and C_1 is

$$C_{BC} = C' + C_1 = 100 + 100 = 200\ \text{pF}$$

The entire circuit now reduces to two capacitors C_4 and C_{BC} (= 200 pF) in series.

$\therefore$ Equivalent capacitance of the network is

$$C = \frac{C_4 \times C_{BC}}{C_4 + C_{BC}} = \frac{100 \times 200}{100 + 200} = \mathbf{\frac{200}{3}\,pF}$$

Charges and p.d. on various Capacitors

$$\text{Total charge, } q = CV = \left(\frac{200}{3} \times 10^{-12}\right) \times 300 = 2 \times 10^{-8}\ \text{C}$$

$\therefore$ Charge on $\mathbf{C_4 = 2 \times 10^{-8}\ C}$

$\therefore$ P.D. across C_4, $V_4 = \dfrac{q}{C_4} = \dfrac{2 \times 10^{-8}}{100 \times 10^{-12}} = \mathbf{200\ V}$

P.D. between B and C, V_{BC} $= 300 - 200 = 100$ V

Charge on C_1, q_1 $= C_1 V_{BC} = (100 \times 10^{-12}) \times 100 = \mathbf{10^{-8}\ C}$

P.D. across C_1, V_1 $= V_{BC} = \mathbf{100\ V}$

P.D. across C_2 $=$ P.D. across $C_3 = 100/2 = \mathbf{50\ V}$

Charge on C_2 $=$ Charge on $C_3 =$ Total charge – Charge on C_1

$= (2 \times 10^{-8}) - (10^{-8}) = \mathbf{10^{-8}\ C}$

Example 17: *Fig. 5.16 shows a network of four capacitors. Determine the equivalent capacitance between points A and B. If a 10V battery is connected between A and B, how much total charge will be stored on the capacitors.*

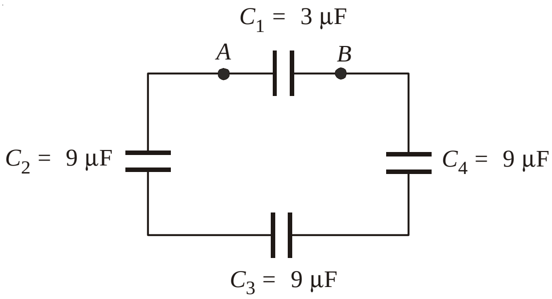


Fig. 5.16

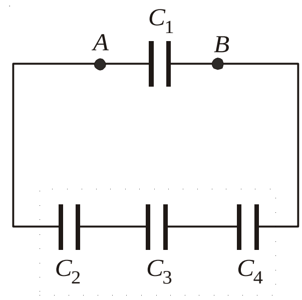


Fig. 5.17

Solution: The given network is equivalent to the network shown in Fig. 5.17. The equivalent capacitance C' of the series connected capacitors C_2, C_3 and C_4 is given by;

$$\frac{1}{C'} = \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9} = \frac{1}{3}$$

$\therefore$ $C' = 3\,\mu\text{F}$

Between points A and B, we now have capacitors C_1 and C' (= 3 μF) in parallel. Therefore, the equivalent capacitance C_{AB} between points A and B of the network is

$$C_{AB} = C_1 + C' = 3 + 3 = \mathbf{6\,\mu F}$$

Total charge stored on the capacitors is

$$q = C_{AB} \times V = (6 \times 10^{-6}) \times 10 = 60 \times 10^{-6}\,\text{C} = \mathbf{60\,\mu C}$$

Example 18: Calculate the equivalent capacitance between points A and B in Fig. 5.18.

Solution: Refer to Fig. 5.18. It is clear that one plate of each capacitor is connected to point A (plate 1, plate 4, and plate 5). Similarly, other plate of each capacitor (plate 2, plate 3 and plate 6) is connected to point B. Therefore, the three capacitors are in parallel. Hence, equivalent capacitance between A and B is

$$C_{AB} = C_1 + C_2 + C_3$$

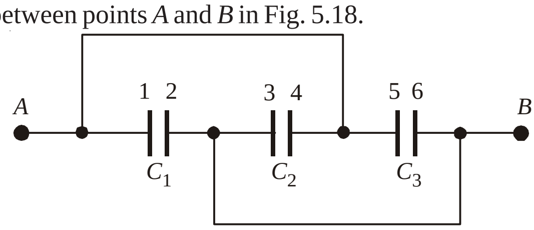


Fig. 5.18

Example 19: *In the circuit shown in Fig. 5.19, find (i) the equivalent capacitance between A and D and (ii) the charge on 12 μF capacitor.*

Solution:

(*i*) $C_{AB} = 10\,\mu\text{F}$

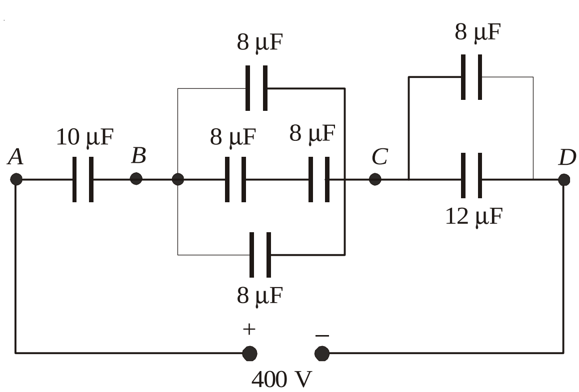


Fig. 5.19

$$C_{BC} = \left(\frac{8 \times 8}{8 + 8}\right) + (8) + (8) = 4 + 8 + 8 = 20\,\mu\text{F}$$

$$C_{AC} = \frac{C_{AB} \times C_{BC}}{C_{AB} + C_{BC}} = \frac{10 \times 20}{10 + 20} = \frac{20}{3}\,\mu\text{F}$$

$$C_{CD} = 8 + 12 = 20\,\mu\text{F}$$

$\therefore$ $$C_{AD} = \frac{(20/3) \times 20}{(20/3) + 20} = \mathbf{5\,\mu F}$$

(*ii*) Total charge, $q = C_{AD} \times V = (5 \times 10^{-6}) \times 400 = 0.002\text{C}$

The total charge will divide between the parallel capacitors connected in the branch *CD*.

P.D. between *C* and *D*, $V_{CD} = \dfrac{q}{C_{CD}} = \dfrac{0.002}{20 \times 10^{-6}} = 100\text{ V}$

∴ Charge on 12 μF capacitor = $(12 \times 10^{-6}) \times 100 = 1.2 \times 10^{-3}\text{C} = \mathbf{1.2\ \mu C}$

Example 20: *In the network shown in Fig. 5.20 (i), $C_1 = C_2 = C_3 = C_4 = 8$ μF and $C_5 = 10$ μF. Find the equivalent capacitance between points A and B.*

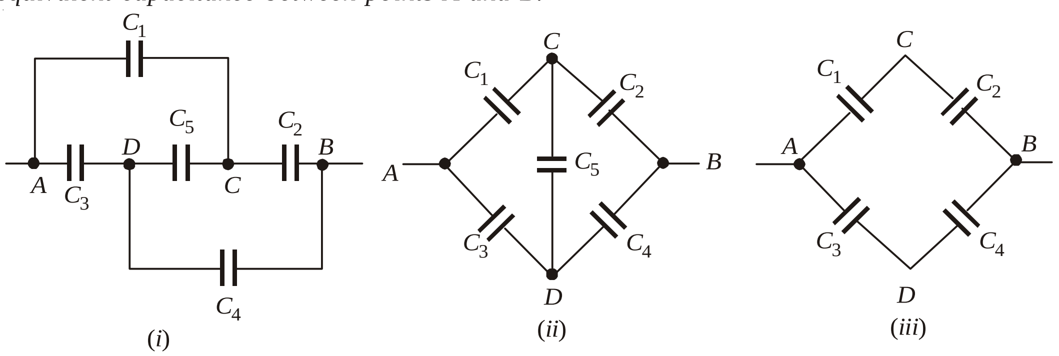


Fig. 5.20

Solution: A little reflection shows that circuit of Fig. 5.20 (*i*) can be drawn as shown in Fig. 5.20 (*ii*). We find that the circuit is a Wheatstone bridge. Since the product of opposite arms of the bridge are equal ($C_1C_4 = C_2C_3$ because $C_1 = C_2 = C_3 = C_4$), the bridge is balanced. It means that points *C* and *D* are at the same potential. Therefore, there will be no charge on capacitor C_5. Hence, this capacitor is ineffective and can be removed from the circuit as shown in Fig. 5.20 (*iii*). Referring to Fig. 5.20 (*iii*), the equivalent capacitance *C′* of the series connected capacitors C_1 and C_2 is

$$C' = \frac{C_1 C_2}{C_1 + C_2} = \frac{8 \times 8}{8 + 8} = 4\mu\text{F}$$

The equivalent capacitance *C″* of series connected capacitors C_3 and C_4 [See Fig. 5.20 (*iii*)].

$$C'' = \frac{C_3 C_4}{C_1 + C_2} = \frac{8 \times 8}{8 + 8} = 4\mu\text{F}$$

Now $C_{AB} = \text{C}' \parallel \text{C}'' = 4 \parallel 4 = 4 + 4 = \mathbf{8\ \mu F}$

Example 21: *A mica dielectric parallel plate capacitor has 21 plates, each having an effective area of 5 cm² and each separated by a gap of 0.005mm. Find the capacitance of the capacitor. Take the relative permittivity of mica as 6.*

Solution: For a multiplate capacitor, capacitance is given by;

$$C = (n-1)\frac{\varepsilon_0 K A}{d}$$

$$= (21-1)\frac{(8.854 \times 10^{-12}) \times 6 \times (5 \times 10^{-4})}{0.005 \times 10^{-3}}$$

$= 0.1062 \times 10^{-6}\text{ F} = \mathbf{0.1062\ \mu F}$

Example 22: *A parallel plate capacitor has three similar parallel plates. Find the ratio of capacitance when the inner plate is mid-way between the outers to the capacitance when inner plate is three times as near one plate as the other.*

Solution: Fig. 5.21 (*i*). shows the condition when the inner plate is mid-way between the outer plates. The arrangement is equivalent to two capacitors in parallel. Capacitance of this capacitor,

$$C_1 = \frac{\varepsilon_0 K A}{d/2} + \frac{\varepsilon_0 K A}{d/2} = \frac{4\varepsilon_0 K A}{d}$$

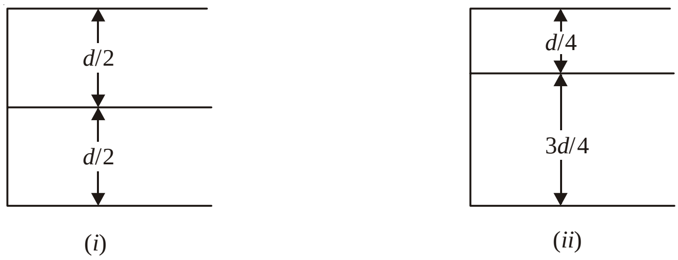


Fig. 5.21

Fig. 5.21 (*ii*) shows the condition when the inner plate is three times as near as one plate as the other.

Capacitance C_2 of this capacitor,

$$C_2 = \frac{\varepsilon_0 K A}{d/4} + \frac{\varepsilon_0 K A}{3d/4} = \frac{16\varepsilon_0 K A}{3d}$$

$\therefore$ $C_1/C_2 = \mathbf{0.75}$

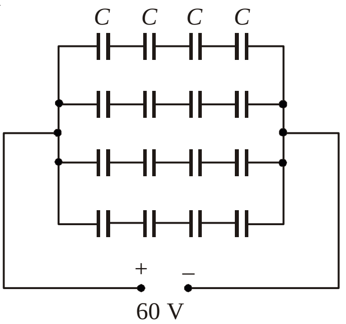


Fig. 5.22

Example 23: *Given some capacitors of 0.1μF capable of withstanding 15V. Calculate the number of capacitors needed if it is desired to obtain a capacitance of 0.1μF for use in a circuit involving 60V.*

Solution:

Capacitance of each capacitor, $C = 0.1$ μF

Voltage rating of each capacitor, $V_C = 15$ V

Supply voltage, $V = 60$ V

Since each capacitor can withstand 15 V, the number of capacitors to be connected in series = 60/15 = 4. Capacitance of 4 series-connected capacitors, $C_S = C/4 = 0.1/4 = 0.025$ μF. Since it is desired to have a total capacitance of 0.1 μF, number of such rows in parallel = $C/C_S = 0.1/0.025 = 4$.

$\therefore$ Total number of capacitors = 4 × 4 = **16**

Fig. 5.22 shows the arrangement of capacitors.

Example 24: *A cylindrical capacitor has two co-axial cylinders of length 20 cm and radii 15.4 cm and 15 cm respectively. The relative permittivity of the insulation is 5. If a p.d. of 5000V is maintained between the two cylinders, determine (i) capacitance of cylindrical capacitor and (ii) potential of inner cylinder.*

Solution:

(*i*) The capacitance of a cylindrical capacitor is

$$C = \frac{K l}{41.4 \log_{10}(b/a)} \times 10^{-9} \text{ F} = \frac{5 \times 0.2}{41.4 \log_{10}(15.4/15)} \times 10^{-9}$$

$$= \mathbf{2.11 \times 10^{-9} F}$$

(*ii*) Since the outer cylinder is earthed, the potential of the inner cylinder is equal to p.d. between the two cylinders, *i.e.*, potential of inner cylinder = **5000V.**

Example 25: *A 5μF capacitor is charged to a p.d. of 100V and then connected to an uncharged 3 μF capacitor. Calculate p.d. across the capacitors.*

Solution: Charge on 5 μF capacitor, $q = CV = (5 \times 10^{-6}) \times 100 = 0.0005$ C

When the two capacitors are connected through a wire, the total capacitance $C_P = 5 + 3 = 8$ μF. The charge 0.0005 C is distributed between the two capacitors to have a common p.d.

$$\therefore \text{ P.D. across capacitors } = \frac{q}{C_P} = \frac{0.0005}{8 \times 10^{-6}} = \mathbf{62.5\ V}$$

Example 26: *Two capacitors of capacitance 4 μF and 6 μF respectively are connected in series across a p.d. of 250V. The capacitors are disconnected from the supply and are reconnected in parallel with each other. Calculate the new p.d. and charge on each capacitor.*

Solution: In series-connected capacitors, p.d.s across the capacitors are in the inverse ratio of their capacitances.

$$\therefore \text{ P.D. across 4 μF capacitor } = 250 \times \frac{6}{4+6} = 150 \text{ V}$$

Charge on 4 μF capacitor = $(4 \times 10^{-6}) \times 150 = 0.0006$ C

Since the capacitors are connected in series, charge on each capacitor is the same.

$\therefore$ Charge on both capacitors = 2 × 0.0006 = 0.0012 C

Parallel connection. When the capacitors are connected in parallel, the total capacitance $C_P = 4 + 6 = 10$ μF. The total charge 0.0012 C is distributed between the capacitors to have a common p.d.

$$\therefore \text{ P.D. across capacitors } = \frac{\text{Total charge}}{C_P} = \frac{0.0012}{10 \times 10^{-6}} = \mathbf{120\ V}$$

Charge on 4 μF capacitor = $(4 \times 10^{-6}) \times 120 = 480 \times 10^{-6}$ = **480 μC**

Charge on 6 μF capacitor = $(6 \times 10^{-6}) \times 120 = 720 \times 10^{-6}$ = **720 μC**

Example 27: *Two capacitors A and B are connected in series across a 200V d.c. supply. The p.d. across A is 120V. This p.d. is increased to 140V when a 3μF capacitor is connected in parallel with B. Calculate the capacitance of A and B.*

Solution: Let C_1 and C_2 μF be the capacitances of the capacitors A and B respectively. When the capacitors are connected in series [See Fig. 5.31 (*i*)], charge on each capacitor is the same.

$\therefore \quad C_1 \times 120 = C_2 \times 80$

or $\quad C_2 = 1.5\,C_1$ ...(*i*)

When a 3 μF capacitor is connected in parallel with *B* [See Fig. 5.31 (*ii*)], the combined capacitance of this parallel branch is $(C_2 + 3)$. Thus the circuit shown in Fig. 5.31 (*ii*) can be thought of as a series circuit consisting of capacitances C_1 and $(C_2 + 3)$ connected in series.

$\therefore \quad C_1 \times 140 = (C_2 + 3)\,60$

or $\quad 7C_1 - 3C_2 = 9$ ...(*ii*)

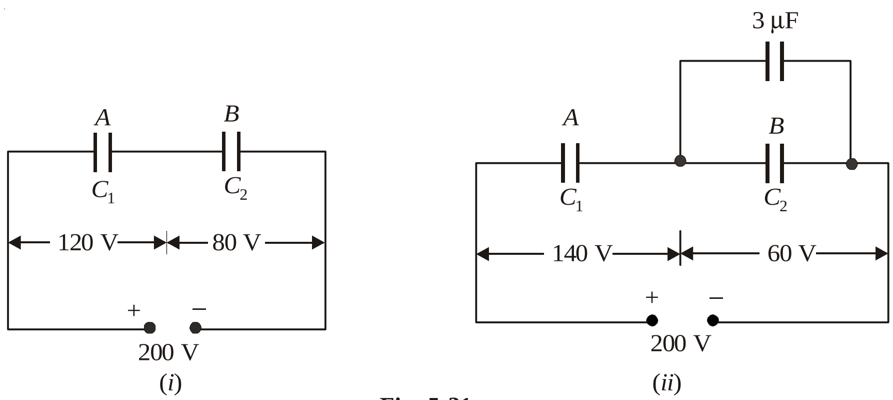


Fig. 5.31

Solving eqs. (*i*) and (*ii*), we have, C_1 = **3.6 μF**; C2 = **5.4 μF**

Example 28: *A 16 μF capacitor is charged to 100V. After being disconnected, it is immediately connected to an uncharged capacitor of 4 μF. Determine (i) the p.d. across the combination (ii) the electrostatic energies before and after the capacitors are connected.*

Solution: $C_1 = 16\,\mu F;\ C_2 = 4\,\mu F$

Before joining

Charge on 16 μF capacitor, $q = C_1V_1 = (16 \times 10^{-6}) \times 100 = 1.6 \times 10^{-3}$ C

Energy stored, $U_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2}(16 \times 10^{-6}) \times 100^2 = \mathbf{0.08\ J}$

After joining

When the capacitors are connected through a wire, the total capacitance, $C_p = C_1 + C_2 = 16 + 4 = 20$ μF. The charge 1.6×10^{-3} C distributes between the two capacitors to have a common p.d. of *V* volts.

$\therefore$ P.D. across capacitors, $V = \frac{q}{C_P} = \frac{1.6 \times 10^{-3}}{20 \times 10^{-6}} = \mathbf{80\ V}$

Energy stored, $U_2 = \frac{1}{2}C_PV^2 = \frac{1}{2}(20 \times 10^{-6}) \times (80)^2 = \mathbf{0.064\ J}$

It may be noted that there is a loss of energy. This is primarily due to the heat dissipated in the conductor connecting the capacitors.

Example 29: *A metal sphere 4 m in diameter is charged to a potential of 3 MV. Calculate the heat generated when the sphere is earthed through a long resistance wire.*

Solution: Potential at the surface of sphere, $V = 9 \times 10^9 \frac{q}{r}$

$\therefore$ Charge on sphere, $q = \frac{V \times r}{9 \times 10^9} = \frac{(3 \times 10^6) \times 2}{9 \times 10^9} = 0.67 \times 10^{-3}$ C

Energy stored in sphere $= \frac{1}{2}qV = \frac{1}{2}(0.67 \times 10^{-3}) \times (3 \times 10^6) = \mathbf{1005\ J}$

When the sphere is earthed, stored energy will be dissipated as heat in the resistance wire.

Example 30. *A parallel plate capacitor is connected to a 12 V battery. The charge on the capacitor is 1.35×10^{-10} C. If the plate separation is decreased to half, find the extra charge given by the battery.*

Solution. When the plate separation is decreased to half, the capacitance ($C = \varepsilon_0 A/d$) becomes twice. Therefore, the charge on the capacitor ($q' = C'V$)becomes twice. The extra charge is supplied by the battery.

Extra charge supplied by battery = $q' - q = 2q - q = q$ = $\mathbf{1.35 \times 10^{-10}}$ **C**

Example 31. *A parallel plate 100 μF capacitor is charged to 500V. If the distance between the plates is halved, what will be the new potential difference between the plates and what will be the change in the new stored energy*?

Solution:

$C = 100\mu F = 100 \times 10^{-6} F = 10^{-4} F; \quad V = 500$ volts

When plate separation is decreased to half, the new capacitance C' becomes twice *i.e.* $C' = 2C$. Since the capacitor is not connected to the battery, the charge on the capacitor remains the same. The potential difference between the plates must decrease to maintain the same charge.

$$\therefore \quad q = CV = C'V' \quad \text{or} \quad V' = \frac{CV}{C'} = \frac{CV}{2C} = \frac{V}{2} = \frac{500}{2} = \textbf{250 volts}$$

$$\text{New stored energy} = \frac{1}{2}C'V'^2 = \frac{1}{2}(2C)\left(\frac{V}{2}\right)^2$$

$$= \frac{1}{2}\frac{CV^2}{2} = \frac{1}{2}\left(\frac{1}{2}CV^2\right)$$

$$= \frac{1}{2}\left[\frac{1}{2} \times 10^{-4} \times (500)^2\right] = \textbf{6.25 J}$$

Example 32. *Fig. 5.32 shows a circuit for a camera flash. A 2000 μF capacitor is charged by 1.5V cell. When a flash is required, the energy stored in the capacitor is made to discharge through a discharge tube in 0.1 ms giving a powerful flash. Calculate the energy stored in the capacitor and power of the flash.*

Solution:

Energy stored in the capacitor is

$$U = \frac{1}{2}CV^2 = \frac{1}{2} \times (2000 \times 10^{-6}) \times (1.5)^2$$

$$= \mathbf{2.25 \times 10^{-3}\ J}$$

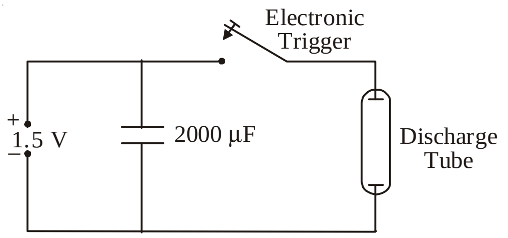


Fig. 5.32

In order to produce the flash, the capacitor discharges in 0.1 ms ($= 0.1 \times 10^{-3}$ s).

$$\therefore \text{ Power of flash} = \frac{U}{\text{Time}} = \frac{2.25 \times 10^{-3}}{0.1 \times 10^{-3}} = \textbf{22.5 W}$$

Example 33: A *parallel plate capacitor is partially filled with an ebonite plate of thickness 6 mm. The area of the plates of the capacitor is 2 × 10^{-3} m^2 and the distance between them is 0.01m. The dielectric constant for ebonite is 3. Calculate the capacitance of the capacitor.*

Solution:

$$\text{Capacitance of the capacitor, } C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$$

Here $\quad A = 2 \times 10^{-3}\ \text{m}^2;\ d = 0.01\ \text{m};\ t = 6 \times 10^{-3}\ \text{m};\ K = 3$

$$\therefore \quad C = \frac{(8.854 \times 10^{-12}) \times 2 \times 10^{-3}}{0.01 - 6 \times 10^{-3}(1 - 1/3)} = \mathbf{2.95 \times 10^{-12}\ F}$$

Example 34: *A parallel plate capacitor has plate area of 2 m^2 spaced by three layers of different dielectric materials. The relative permittivities are 2, 4, 6 and thicknesses are 0.5, 1.5 and 0.3 mm respectively. Calculate the capacitance of the capacitor.*

Solution: Capacitance of capacitor, $C = \dfrac{\varepsilon_0 A}{\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \dfrac{t_3}{K_3}}$

$$= \frac{(8.854 \times 10^{-12}) \times 2}{\dfrac{0.5 \times 10^{-3}}{2} + \dfrac{1.5 \times 10^{-3}}{4} + \dfrac{0.3 \times 10^{-3}}{6}} = \mathbf{0.026 \times 10^{-6}\ F}$$

Example 35: *A capacitor is composed of two plates separated by 3 mm dielectric of relative permittivity 4. An additional piece of insulation 5 mm thick is now inserted between the plates. If the capacitor has now capacitance one-third of its original capacitance, find the relative permittivity of the additional dielectric.*

Solution: Fig. 5.40 (*i*) and Fig. 5.40 (*ii*) respectively show the two cases.

For the first case, $\quad C = \dfrac{\varepsilon_0 K_1 A}{d} = \dfrac{\varepsilon_0 \times 4 \times A}{d}$ ...(*i*)

For the second case, $\quad \dfrac{C}{3} = \dfrac{\varepsilon_0 A}{\dfrac{t_1}{K_1} + \dfrac{t_2}{K_2}} = \dfrac{\varepsilon_0 A}{\dfrac{3 \times 10^{-3}}{4} + \dfrac{5 \times 10^{-3}}{K_2}}$ ...(*ii*)

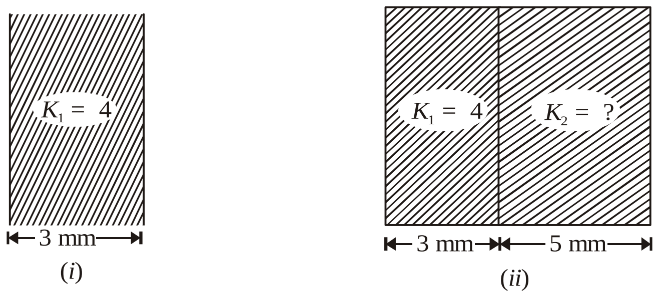


Fig. 5.40

Dividing eq. (*i*) by eq. (*ii*), $3 = \frac{4}{3}\left(\frac{3}{4} + \frac{5}{K_2}\right)$

$\therefore$ $K_2 = \mathbf{3.33}$

Example 36: *An air capacitor has two parallel plates of 1500 cm² area and held 5 mm apart. If a dielectric slab of area 1500 cm², thickness 2 mm and relative permittivity 3 is now introduced between the plates, what must be the new separation between the plates to bring the capacitance to the original value.*

Solution: When a dielectric slab of thickness t ($t < d$) is introduced between the plates of air capacitor, the capacitance is given by;

$$C = \frac{\varepsilon_0 A}{d - (t - t/K)} \qquad \text{...(i)}$$

If the medium were totally air, the capacitance would have been

$$C_0 = \frac{\varepsilon_0 A}{d} \qquad \text{...(ii)}$$

An inspection of eqs. (*i*) and (*ii*) reveals that with the introduction of the dielectric slab between the plates of air capacitor, its capacitance increases. The distance between the plates is effectively reduced by $t - (t/K)$. In order to bring the capacitance to the original value, the plates must be separated by this much distance in air.

$\therefore$ New separation between plates $= d + (t - t/K) = 5 + (2 - 2/3) = \mathbf{6.33\ mm}$

Example 37: *The capacitance of a parallel plate capacitor is 50 pF and the distance between the plates is 4 mm. It is charged to 200V and the charging battery is removed. Now a dielectric slab (K = 4) of thickness 2 mm is introduced between the plates. Find (i) final charge on each plate, (ii) p.d. between the plates, (iii) final energy in the capacitor, and (iv) energy loss.*

Solution: The capacitance of parallel plate air capacitor is given by;

$$C_0 = \frac{\varepsilon_0 A}{d} \qquad \text{...(i)}$$

When a dielectric slab of thickness t is placed between the plates of the capacitor, capacitance is given by;

$$C_m = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}} \qquad \text{...(ii)}$$

$$\therefore \quad \frac{C_0}{C_m} = \frac{d - t + (t/K)}{d} = \frac{4 - 2 + (2/4)}{4} = \frac{5}{8}$$

Before Introduction of the Dielectric Slab

When air capacitor is charged to 200 V and then battery is removed, charge (q) on each plate is

$$q = C_0 \times V_0 = (50 \times 10^{-12}) \times 200 = 10^{-8}\,\text{C}$$

After Introduction of the Dielectric Slab

(*i*) Since the battery is removed before the introduction of the dielectric slab, the charge on capacitor plates will remain the same after the introduction of the dielectric slab.

Final charge on each plate $= q = \mathbf{10^{-8}\ C}$

(*ii*) When dielectric slab is placed between the plates of the capacitor, its capacitance increases to C_m and p.d. between plates decreases to V.

$$q = C_0 V_0 = C_m V$$

$$\therefore \quad V = \frac{C_0}{C_m} V_0 = \left(\frac{5}{8}\right) \times 200 = \mathbf{125\ V}$$

Note that p.d. between the plates decreases. This is in agreement with theory.

(*iii*) Final energy stored in the capacitor,

$$U = \frac{1}{2}qV = \frac{1}{2}(10^{-8}) \times 125 = \mathbf{6.25 \times 10^{-7}\ J}$$

(*iv*) Initial stored energy, $U_0 = \frac{1}{2}qV_0$; Final stored energy, $U = \frac{1}{2}qV$

$\therefore$ Energy loss $= U_0 - U = \frac{1}{2}q(V_0 - V)$

$$= \frac{1}{2}(10^{-8})(200 - 125) = \mathbf{3.75 \times 10^{-7}\ J}$$

This loss of energy will be apparent to the person who introduced the slab. The capacitor would exert a tiny force on the slab and would do work on it equal to 3.75×10^{-7} J.

Note: Initial stored energy, $U_0 = \frac{1}{2}qV_0 = \frac{1}{2}(10^{-8}) \times 200 = 10^{-6}$ J

Final stored energy $U = 6.25 \times 10^{-7}$ J

Had the slab been 4 mm thick (= distance between plates), the energy stored in the capacitor would have been smaller by $1/K$ *i.e.* U_0/K.

Thus we arrive at a very important conclusion that final energy after the slab filling entire space is introduced is smaller by $1/K$.

Example 38. *A parallel plate capacitor having plate separation of 3mm possesses a capacitance of 17.7 pF. The capacitor is connected to a 100V supply. Explain what would happen if a 3 mm thick mica sheet of dielectric constant 6 were inserted between the plates (i) while the voltage supply remains connected (ii) after the supply was disconnected.*

Solution. Capacitance of capacitor, $C = 17.7$ pF ; Dielectric constant of mica, $K = 6$

(*i*) **When voltage supply remains connected**

When mica sheet is inserted between the plates of the capacitor, the capacitance becomes K times *i.e.*

$$C' = KC = 6 \times 17.7 = \mathbf{106.2\ pF}$$

The p.d. between the plates of the capacitor remains equal to **100 V.**

Since $C = q/V$ and V is same but C has increased, the charge on capacitor must increase *i.e*. Charge on capacitor, $q' = CV = 106.2 \times 10^{-12} \times 100 = \mathbf{1.06 \times 10^{-8}\ C}$

The extra charge is supplied by the battery.

(*ii*) **After the voltage supply is disconnected**

$$C' = KC = 6 \times 17.7 = \mathbf{106.2\ pF}$$

Charge on capacitor, $q = CV = 17.7 \times 10^{-12} \times 100 = \mathbf{1.77 \times 10^{-9}\ C}$

Since the supply is disconnected, the charge on the plates remains the same. Because the capacitance ($C = q/V$) has increased, the potential difference across the plates must decrease to maintain the same charge.

$$V' = \frac{q}{C'} = \frac{1.77 \times 10^{-9}}{106.2 \times 10^{-12}} = \mathbf{16.67\ V}$$

Example 39. *In a Van de Graaff generator, the shell electrode is at 25 × 105 V. The dielectric strength of the gas surrounding the electrode is 5 × 10^7 V/m. Calculate the minimum radius of the spherical shell.*

Solution. Electric potential V of a charged shell is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

Electric field at the surface of a charged shell is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\therefore \quad \frac{V}{E} = r \quad \text{or} \quad r = \frac{V}{E} = \frac{25 \times 10^5}{5 \times 10^7} = 5 \times 10^{-2}\ \text{m} = \mathbf{5\,cm}$$

MORE NUMERICAL PROBLEMS

1. *A capacitor of 20 μF and charged to 500 V is connected in parallel with another capacitor of 10 μF charged to 200 V. Find the common potential.* **[Roorkee]**
[400 V]

Hint. Charge on one capacitor, $q_1 = C_1 V_1 = (20 \times 10^{-6}) \times 500 = 0.01$ C

Charge on second capacitor, $q_2 = C_2 V_2 = (10 \times 10^{-6}) \times 200 = 0.002$ C

Total charge on capacitors, $q = q_1 + q_2 = 0.01 + 0.002 = 0.012$ C

Total capacitance, $C = C_1 + C_2 = (20 \times 10^{-6}) + (10 \times 10^{-6}) = 30 \times 10^{-6}$ F

$$\therefore \quad \text{Common potential, } V = \frac{q}{C} = \frac{0.012}{30 \times 10^{-6}} = 400 \text{ V}$$

2. *Five equal capacitors connected in series have a resultant capacitance of 4 μF. What is the total energy stored in these when connected in parallel and charged to 400 V ?*
[E.A.M.C.E.T. 91] [8 J]

Hint. Suppose C μF is the capacitance of each capacitor. Since 5 (= n) capacitors are connected in series,

$$\frac{C}{n} = 4 \text{ or } C = 4\,n = 4 \times 5 = 20 \text{ μF}$$

When the capacitors are connected in parallel, then equivalent capacitance C' is

$$C' = 5 \times 20 = 100 \text{ μF} = 100 \times 10^{-6} \text{ F}$$

Energy stored is given by;

$$U = \frac{1}{2} C' V^2 = \frac{1}{2} \times (100 \times 10^{-6}) \times (400)^2 = 8 \text{ J}$$

3. *Find the length of the paper used in a capacitor of capacitance 2 μF if the dielectric constant of the paper is 2.5 and its width and thickness are 50 mm and 0.05 mm respectively.* **[90 m]**

Hint. $$C = \frac{\varepsilon_0 K A}{d}$$

Here $C = 2 \text{ μF} = 2 \times 10^{-6}$ F; $K = 2.5$; $d = 0.05$ mm $= 0.05 \times 10^{-3}$ m

$$\therefore \quad A = \frac{Cd}{\varepsilon_0 K} = \frac{(2 \times 10^{-6}) \times (0.05 \times 10^{-3})}{8.85 \times 10^{-12} \times 2.5} = 4.5 \text{ m}^2$$

$$\therefore \quad \text{Length} = \frac{\text{Area}}{\text{Width}} = \frac{4.5}{50 \times 10^{-3}} \text{ m} = 90 \text{ m}$$

4. *A 5 μF capacitor is fully charged across a 12 V battery and connected to an uncharged capacitor. The voltage across it is found to be 3 V. What is the capacity of the uncharged capacitor ?*
[E.A.M.C.E.T.] [15 μF]

Hint. The common potential V after connection is

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Here $C_1 = 5$ μF; $V = 3$ volts; $V_1 = 12$ volts, $V_2 = 0$; $C_2 = ?$

$$\therefore \quad 3 = \frac{5 \times 12 + C_2 \times 0}{5 + C_2}$$

$$\therefore \quad C_2 = 15 \text{μF}$$

5. *A parallel-plate capacitor has plates of dimensions 2 cm × 3 cm. The plates are separated by a 1 mm thickness of paper.*

(i) Find the capacitance of the paper capacitor. The dielectric constant of paper is 3.7.

(ii) What is the maximum charge that can be placed on the capacitor ? The dielectric strength of paper is 16 × 16^6 *V/m.* **[(*i*) 19.6 × 10^{-12} F (*ii*) 0.31 μC]**

Hint. (*i*) $$C = \frac{\varepsilon_0 K A}{d}$$

Here $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2 N^{-1} m^{-2}; $K = 3.7$; $A = 6\ \ 10^{-4}$ m^2; $d = 1\ \ 10^{-3}$ m

$$C = \frac{(8.85 \times 10^{-12}) \times (3.7) \times (6 \times 10^{-4})}{1 \times 10^{-3}} = 19.6 \times 10^{-12} \text{ F}$$

(*ii*) Since the thickness of the paper is 1 mm, the maximum voltage that can be applied before break-down occurs is

$$V_{max} = E_{max} \times d$$

Here $E_{max} = 16 \times 10^6$ V/m; $d = 1$ mm $= 1 \times 10^{-3}$ m

$\therefore$ Maximum charge that can be placed on capacitor is

$$q_{max} = C\,V_{max} = (19.6 \times 10^{-12}) \times (16 \times 10^3) = 0.31 \times 10^{-6}\,\text{C} = 0.31\,\mu\text{C}$$

6. *A capacitor of capacitance $C_1 = 1\ \mu F$ withstands the maximum voltage $V_1 = 6$ kV while another capacitance $C_2 = 2\ \mu F$ withstands the maximum voltage $V_2 = 4$ kV. What maximum voltage will the system of these two capacitors withstand if they are connected in series ?*

[M.N.R. 1992] **[9 kV]**

Hint. The maximum charge q_1 and q_2 that can be placed on C_1 and C_2 are

$$q_1 = C_1 V_1 = (1 \times 10^{-6}) \times (6 \times 10^3) = 6 \times 10^{-3}\,\text{C}$$

$$q_2 = C_2 V_2 = (2 \times 10^{-6}) \times (4 \times 10^3) = 8 \times 10^{-3}\,\text{C}$$

The charge on capacitor C_1 should not exceed 6×10^{-3} C. Therefore, when capacitors are connected in series, the maximum charge that can be placed on the capacitors is 6×10^{-3} C ($= q_1$).

$$\therefore \quad V_{max} = \frac{q_1}{C_1} + \frac{q_1}{C_2} = \frac{6 \times 10^{-3}}{1 \times 10^{-6}} + \frac{6 \times 10^{-3}}{2 \times 10^{-6}}$$

$$= 6 \times 103 + 3 \times 10^3 = 10^3\,(6 + 3) = 9 \times 10^3\,\text{V} = 9\,\text{kV}$$

7. *A parallel-plate capacitor is charged with a battery to a charge q_0 as shown in Fig. 5.70 (i). The battery is then removed and the space between the plates is filled with a dielectric of dielectric constant K. Find the energy stored in the capacitor before and after the dielectric is inserted.* $\left[\dfrac{q_0^2}{2C_0}; \dfrac{q_0^2}{2KC_0}\right]$

Hint. Energy stored in the capacitor in the absence of dielectric is

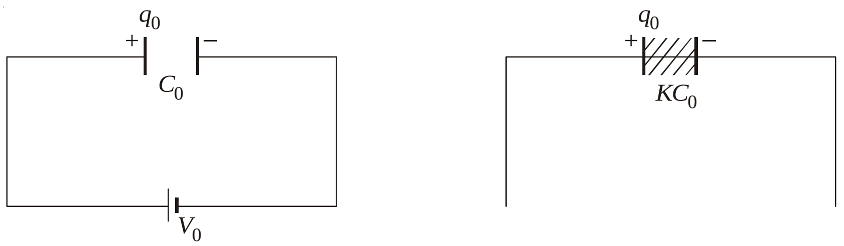
$$U_0 = \frac{1}{2} C_0 V_0^2$$

Since $V_0 = q_0/C_0$, this can be expressed as :

$$U_0 = \frac{q_0^2}{2C_0} \qquad \text{... }(i)$$

Eq. (*i*) gives the energy stored in the capacitor in the absence of dielectric.

After the battery is removed and the dielectric is inserted between the plates, *charge on the capacitor remains the same*. But the capacitance of the capacitor is increased K times *i.e.*, new capacitance is $C' = K C_0$ [See Fig 5.70 (*ii*)].



(*i*) (*ii*)

Fig. 5.70

$\therefore$ Energy stored in the capacitor after insertion of dielectric is

$$U = \frac{q_0^2}{2C'} = \frac{q_0^2}{2KC_0} = \frac{U_0}{K}$$

or $$U = \frac{U_0}{K} \qquad \text{... }(ii)$$

Since $K > 1$, we find that final energy is *less* than the initial energy by the factor $1/K$. How will you account for "missing energy" ? When the dielectric is inserted into the capacitor, it gets pulled into the device. The external agent must do negative work to keep the dielectric from accelerating. This work is simply $= U_0 - U$. Alternately, the positive work done by the system $= U_0 - U$.

8. *Suppose in the above problem, the capacitor is kept connected with the battery and then dielectric is inserted between the plates. What will be the change in charge, the capacitance, the potential difference, the electric field and the stored energy ?*

Hint. Since the battery remains connected, the potential difference V_0 will remain unchanged. As a result, electric field ($= V_0/d$) will also remain unchanged.

The capacitance C_0 will increase to $C = K C_0$

The charge will also increase to $q = K q_0$ as explained below.

$$q_0 = C_0 V_0 \; ; q = C V_0 = KC_0 V_0 = K q_0$$

Initial stored energy, $U_0 = \frac{1}{2} C_0 V_0^2$

Final stored energy, $U_0 = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} K C_0 V_0^2 = KU_0$

$\therefore$ $U = KU_0$

Note that stored energy is increased K times. Will any work be done in inserting the dielectric? The answer is yes. In this case, the work will be done by the battery. The battery not only gives the increased energy to the capacitor but also provides the necessary energy for inserting the dielectric.

9. *A parallel plate capacitor is maintained at a certain potential difference. When a 3mm slab is introduced between the plates, in order to maintain the same potential difference, the distance between the plates is increased by 2.4 mm. Find the dielectric constant of the slab.*

[M.N.R.] **[$K = 5$]**

Hint. The capacitance of parallel-plate capacitor in air is

$$C = \frac{\varepsilon_0 A}{d} \quad \text{... (i)}$$

With the introduction of slab of thickness t, the new capacitance is

$$C' = \frac{\varepsilon_0 A}{d' - t(1 - 1/K)} \quad \text{... (ii)}$$

Now the charge ($q = CV$) remains the same in the two cases.

$\therefore$ $$\frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{d' - t(1 - 1/K)}$$

or $d = d' - t\,(1 - 1/K)$

Here $d' = d + 2.4 \times 10^{-3}$ m ; $t = 3$ mm $= 3 \times 10^{-3}$ m

$\therefore$ $$d = d + 2.4 \times 10^{-3} - 3 \times 10^{-3} \left(1 - \frac{1}{K}\right)$$

or $$2.4 \times 10^{-3} = 3 \times 10^{-3} \left(1 - \frac{1}{K}\right)$$

$\therefore$ $K = 5$

10. *The capacitance of a variable radio capacitor can be changed from 50 pF to 950 pF by turning the dial from 0° to 180°. With dial set at 180°, the capacitor is connected to 400 V battery. After charging, the capacitor is disconnected from the battery and the dial is turned at 0°.*

(*i*) *What is the potential difference across the capacitor when the dial reads 0° ?*

(*ii*) *How much work is required to turn the dial if friction is neglected ?* **[M.N.R.]**

[(*i*) 7600 V (*ii*) 1.37×10^{-3} J]

Hint. (*i*) With dial at 0°, the capacitance of the capacitor is

$$C_1 = 50 \text{ pF} = 50 \times 10^{-12} \text{ F}$$

With dial at 180°, the capacitance of the capacitor is

$$C_2 = 950 \text{ pF} = 950 \times 10^{-12} \text{ F}$$

P.D. across C_2, $V_2 = 400$ V

$\therefore$ Charge on C_2, $q = C_2 V_2 = (950 \times 10^{-12}) \times 400 = 380 \times 10^{-9}$ C

When the battery is disconnected, charge q remains the same. Suppose V_1 is the potential difference across the capacitor when the dial reads 0°.

$\therefore$ $q = C_1 V_1$

or $$V_1 = \frac{q}{C_1} = \frac{380 \times 10^{-9}}{50 \times 10^{-12}} = 7600 \text{ V}$$

(*ii*) Work required = Gain in energy of the capacitor

$$= \frac{1}{2} C_1 V_1^2 - \frac{1}{2} C_2 V_2^2$$

$$= \frac{1}{2} \times 50 \times 10^{-12} \times (7600)^2 - \frac{1}{2} \times 950 \times 10^{-12} \times (400)^2$$

$$= 1.37 \times 10^{-3} \text{ J}$$

11. *A 90 pF capacitor is connected to a 12 V battery and is charged to 12 V. How many electrons are transferred from one plate to the other ?* **[6.9 × 109]**

Hint. $q = CV = (90 \times 10^{-12}) \times (12) = 1.1 \times 10^{-9}$ C

Now $q = ne$

∴ Number of electrons transferred is

$$n = \frac{q}{e} = \frac{1.1 \times 10^{-9}}{1.6 \times 10^{-19}} = 6.9 \times 10^{9}$$

12. *If $C_1 = 20$ μF, $C_2 = 30$ μF and $C_3 = 15$ μF and the insulated plate of C_1 be at a potential of 90 V, one plate of C_3 potential of 90 V, one plate of C_3 being earthed, what is the potential difference between the plates of C_2, three capacitors being connected in series ?* **[20 V]**

Hint. Fig. 5.71 shows the conditions of the problem. The equivalent capacitance of this series combination is given by;

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15} = \frac{3}{20}$$

$$C = \frac{20}{3}\,\mu\text{F} = \frac{20}{3} \times 10^{-6}\ \text{F}$$

P.D. across series combination, $V = 90 - 0 = 90$ V

Total charge, $q = CV = \frac{20}{3} \times 10^{-6} \times 90 = 6 \times 10^{-4}$ C

∴ P.D. across $C_2, V_2 = \frac{q}{C_2} = \frac{6 \times 10^{-4}}{30 \times 10^{-6}} = 20\ \text{V}$

13. *A spherical capacitor has an inner sphere of radius 12 cm and outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 μC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.*

(*i*) *Determine the capacitance of the capacitor.*

(*ii*) *What is the potential of the inner sphere ?*

(*iii*) *Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Why is the capacitance smaller in the latter case ?* **[(*i*) 5.55 × 10⁻⁹ F (*ii*) 4.5 × 102 V (*iii*) 1.33 × 10⁻¹¹ F]**

Hint. (*i*) Capacitance of spherical capacitor is given by;

$$C = 4\pi\varepsilon_0 K \frac{ab}{b-a}$$

Here $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$; $K = 32$; $b = 0.13$ m; $a = 0.12$ m

$$\therefore \quad C = \frac{32}{9 \times 10^9} \times \frac{0.12 \times 0.13}{0.13 - 0.12} = 5.55 \times 10^{-9}\ \text{F}$$

(*ii*) Potential of inner sphere is

$$V = \frac{q}{C} = \frac{2.5 \times 10^{-6}}{5.55 \times 10^{-9}} = 4.5 \times 10^{2}\ \text{V}$$

(*iii*) Capacitance of isolated sphere is

$$= 4\pi\varepsilon_0 r = \frac{0.12}{9 \times 10^9} = 1.33 \times 10^{-11}\ \text{F}$$

Note that capacitance of an isolated sphere is much smaller than that of the concentric spheres. It is because in case of concentric spheres, the total potential is distributed over two spheres and the potential difference between the two spheres becomes smaller. Since capacitance ($C = q/V$) is inversely proportional to potential difference, a spherical capacitor has large capacitance.

14. *N drops of mercury of equal radii and possessing equal charges combine to form a big drop. What is the charge, capacitance and potential of the bigger drop ?*

[(*i*) $Q = Nq$ (*ii*) $C = c\ N^{1/3}$ (*iii*) $V = v\,N^{2/3}$]

Hint. Let q, v and c be the charge, potential and capacitance of the individual small drop. The corresponding quantities for the bigger drop are Q, V and C.

Charge on bigger drop = N × charge on small drop

∴ $Q = Nq$

The capacitance of a spherical drop is proportional to the radius.

$$\therefore \qquad \frac{C}{c} = \frac{R}{r}$$

Since mass is a conserved quantity,

$$\frac{4}{3}\pi R^3 \rho = N \times \frac{4}{3}\pi r^3 \rho$$

$$R = r \times N^{1/3}$$

$$\text{or} \qquad \frac{R}{r} = N^{1/3}$$

$$\therefore \qquad \frac{C}{c} = N^{1/3} \quad \text{or} \quad C = c \times N^{1/3}$$

$$\text{Now} \quad V = \frac{Q}{C} \text{ and } v = \frac{q}{c}$$

$$\therefore \qquad \frac{V}{v} = \left(\frac{Q}{C}\right) \times \left(\frac{c}{C}\right) = (N) \times \left(\frac{1}{N^{1/3}}\right) = N^{2/3}$$

$$\text{or} \qquad V = v \times N^{2/3}$$

15. *An infinite identical capacitors each of capacitance 1 μF are connected as shown in Fig. 5.72. What is the equivalent capacitance between terminals A and B ?* **[2 μF]**

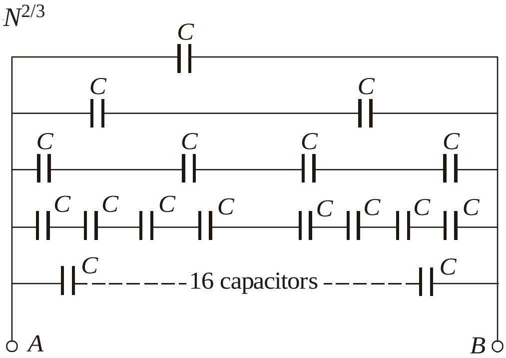


Fig. 5.72

Hint. It is clear from the figure that the rows of capacitors are connected in parallel. The capacitance of first row, second row, third row, fourth row .... is C, $C/2$, $C/4$, $C/8$ .... .

The equivalent capacitance of the arrangement is

$$C_{AB} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + ... = C\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right)$$

$$= C\left[\frac{1}{1 - 1/2}\right] = 2C = 2 \times 1 = 2 \mu\text{F}$$

16. *A spherical capacitor has 10 cm and 12 cm as the radii of inner and outer spheres. The space between the two is filled with a dielectric of dielectric constant 5. Find the capacitance when (i) the outer sphere is earthed and (ii) inner sphere is earthed.*

[(*i*) 3.33 × 10^{-10} F (*ii*) 3.46 × 10^{-10} F]

Hint. (*i*) When the outer sphere is earthed, the capacitance of the spherical capacitor is

$$C = 4\pi\varepsilon_0 K \frac{ab}{b - a}$$

$$\text{Here } 4\pi\varepsilon_0 = \frac{1}{9 \times 10^9} = C^2\,\text{N}^{-1}\,\text{m}^{-2};\ K = 5;\ a = 0.1\text{ m};\ b = 0.12\text{ m}$$

$$\therefore \qquad C = \frac{1}{9 \times 10^9} \times 5 \times \frac{0.1 \times 0.12}{0.12 - 0.1} = 3.33 \times 10^{-10}\text{ F}$$

(*ii*) When the inner sphere is earthed, the system is equivalent to capacitors in parallel. One capacitor is between outer sphere and earth and the other capacitor is between outer sphere and inner earthed sphere. The equivalent capacitance is

$$C = 4\pi\varepsilon_0 b + 4\pi\varepsilon_0 K \frac{ab}{b - a}$$

$$= \frac{1}{9 \times 10^9} \times 0.12 + \frac{1}{9 \times 10^9} \times \frac{0.1 \times 0.12}{0.12 - 0.1}$$

$$= 0.13 \times 10^{-10} + 3.33 \times 10^{-10}$$

$$= 3.46 \times 10^{-10}\text{ F}$$

17. *Find the charge on 5 μF capacitor in the circuit shown in Fig. 5.73.* **[9 μC]**

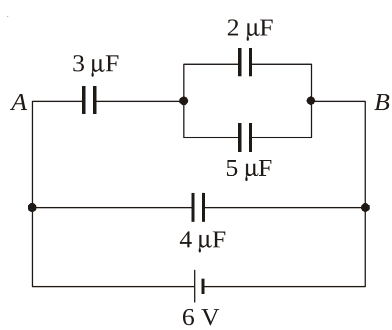


Fig. 5.73

Hint. The p.d. between AB is 6 V. Considering the branch AB, the capacitors 2 μF and 5 μF are in parallel and their equivalent capacitance = 2 + 5 = 7 μF. The branch AB then has 7 μF and

3 μF is series. Therefore, the effective capacitance of branch *AB* is

$$C_{AB} = \frac{7 \times 3}{7 + 3} = \frac{21}{10} \mu\text{F}$$

Total charge in branch *AB*, $q = C_{AB} V = \frac{21}{10} \times 6 = \frac{63}{5}$ μC

P.D. across 3 μF capacitor $= \frac{q}{3} = \frac{63}{5} \times \frac{1}{3} = \frac{21}{5}$ volts

∴ P.D. across parallel combination $= 6 - \frac{21}{5} = \frac{9}{5}$ volts

Charge on 5 μF capacitor $= (5 \times 10^{-6}) \times \frac{9}{5} = 9 \times 10^{-6}$ C = 9 μC

18. *Two parallel plate capacitors A and B having capacitance of 1 μF and 5 μF are charged separately to the same potential of 100 V. Now positive plate of A is connected to the negative plate of B and the negative plate of A is connected to the positive of B. Find the final charge on each capacitor.* **[On *A* = 200/3 μC; On *B* = 1000/3 μC]**

Hint. Initial charge on *A*, $q_1 = C_1 V = (1 \times 10^{-6}) \times 100 = 100$ μC

Initial charge on *B*, $q_2 = C_2 V = (5 \times 10^{-6}) \times 100 = 500$ μC

When the oppositely charged plates of *A* and *B* are connected together, the net charge is

$$q = q_2 - q_1 = 500 - 100 = 400 \text{ μC}$$

$$\text{Final potential difference} = \frac{\text{Net charge}}{\text{Net capacitance}} = \frac{400 \times 10^{-6}}{(1+5)10^{-6}} = \frac{200}{3} \text{V}$$

$$\text{Final charge on } A = C_1 \times \frac{200}{3} = (1 \times 10^{-6}) \times \frac{200}{3} = \frac{200}{3} \text{ μC}$$

$$\text{Final charge on } B = C_2 \times \frac{200}{3} = (5 \times 10^{-6}) \times \frac{200}{3} = \frac{1000}{3} \text{ μC}$$

19. *The radii of the two spherical shells which form an air filled spherical capacitor are 10 cm and 10.5 cm. After the inner shell has been charged to a potential of 100 V, the outer shell is taken apart and removed. What is the final potential of the charged inner shell ?* **[2100 V]**

Hint. The capacitance of air-filled spherical capacitor is

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

Here $4\pi\varepsilon_0 = \frac{1}{9 \times 10^9} = C^2 \text{ N}^{-1} \text{ m}^{-2}$; $a = 0.1$ m; $b = 0.105$ m

$$\therefore \quad C = \frac{1}{9 \times 10^9} \times \frac{0.1 \times 0.105}{0.105 - 0.1} = \frac{2.1}{9 \times 10^9} \text{F}$$

$$\text{Charge on inner sphere, } q = CV = \frac{2.1}{9 \times 10^9} \times 100 = \frac{21}{9} \times 10^{-8} \text{ C}$$

When the outer shell is removed, then charge on the inner sphere remains the same. However, the capacitance will change and is given by

$$C' = 4\pi\varepsilon_0 a = \frac{1}{9 \times 10^9} \times 0.1 = \frac{1}{9} \times 10^{-8} \text{ F}$$

∴ Final potential of the inner charged shell is

$$V' = \frac{q}{C'} = \frac{21}{9} \times 10^{-8} \times \frac{9}{10^{-10}} = 2100 \text{ V}$$

20. *A capacitor is filled with two dielectrics of the same dimensions but of dielectric constants K_1 and K_2 respectively. Find the capacitances in two possible arrangements*

[M.N.R.]. **(*i*) $\frac{2\varepsilon_0 A}{d} \frac{K_1 K_2}{K_1 + K_2}$ (*ii*) $\frac{\varepsilon_0 A}{2d}(K_1 + K_2)$**

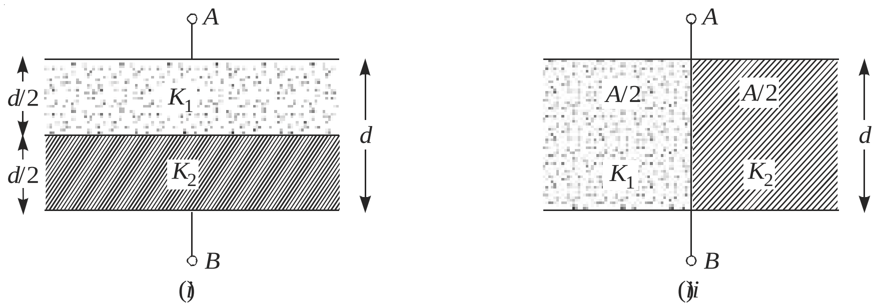


Fig. 5.74

Hint. The two possible arrangements are shown in Fig. 5.74.

(*i*) The arrangement shown in Fig. 5.74 (*i*) is equivalent to two capacitors in series,each with plate area *A* and plate separation *d*/2 *i.e.*,

$$C_1 = \frac{K_1 \varepsilon_0 A}{d/2} = \frac{2K_1 \varepsilon_0 A}{d}$$

$$C_2 = \frac{K_2 \varepsilon_0 A}{d/2} = \frac{2K_2 \varepsilon_0 A}{d}$$

The equivalent capacitance C' is given by;

$$\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d}{2K_1 \varepsilon_0 A} + \frac{d}{2K_2 \varepsilon_0 A} = \frac{d}{2\varepsilon_0 A}\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$$

$$= \frac{d}{2\varepsilon_0 A}\left(\frac{K_1 + K_2}{K_1 K_2}\right)$$

$$\therefore \quad C' = \frac{2\varepsilon_0 A}{d}\left(\frac{K_1 K_2}{K_1 + K_2}\right)$$

(*ii*)The arrangement shown in Fig. 5.74 (*ii*) is equivalent to two capacitors inparallel,each with plate area *A*/2 and plate separation *d i.e.*,

$$C_1 = \frac{K_1 \varepsilon_0 (A/2)}{d} = \frac{K_1 \varepsilon_0 A}{2d}$$

$$C_2 = \frac{K_2 \varepsilon_0 (A/2)}{d} = \frac{K_2 \varepsilon_0 A}{2d}$$

The equivalent capacitance C'' is given by;

$$C'' = C_1 + C_2 = \frac{K_1 \varepsilon_0 A}{2d} + \frac{K_2 \varepsilon_0 A}{2d} = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$

$$\therefore \quad C'' = \frac{\varepsilon_0 A}{2d}(K_1 + K_2)$$